

**Statistical Methods for Artificial Intelligence, Autumn 2006**  
**Problem set 2, Due Wednesday Oct. 11**

Problem 1. (Derivation of the Kalman Filter in one dimension). We use the notation from the wikipedia page on Kalman filters (linked to from the website). Assume that  $x_i$  and  $z_i$  are one dimensional and that  $F = H = 1$  (and there is no input control, i.e.  $B = 0$ ). Here, the variances,  $Q$  and  $R$ , are also one dimensional constants (and not time dependent).

Start by assuming that:

$$\Pr(x_{t-1}|z_1, \dots, z_{t-1}) \sim \mathcal{N}(x_{t-1|t-1}, P_{t-1|t-1})$$

Go through the following steps:

1. Write down the distribution of  $\Pr(x_t|z_1, \dots, z_{t-1})$ . As this is a Gaussian distribution, you only have to specify the mean and variance. (Hint: variances are additive for independent variables).
2. Write down joint distribution of  $\Pr(x_t, z_t|z_1, \dots, z_{t-1})$ . Again, as this is jointly Gaussian, you only need to specify the means and variances. (Hint: just explicitly compute the mean and covariance terms by working out the appropriate expectations).
3. Using this, specify the conditional distribution  $\Pr(x_t|z_1, \dots, z_{t-1}, z_t)$  (again, just specify the mean and covariance). Hint: As we are conditioning on the history  $z_1, \dots, z_{t-1}$ , these can be treated as constants, and you just need to use the equation for how  $P(a, b)$  is related to  $P(a|b)$  for Gaussians (where you should interpret  $b$  as  $z_t$ ). Check the wikipedia page on the multivariate normal distribution, and look at the conditional distributions section.
4. Note that in part 3 you just worked out the update for  $x_{t|t}$  and  $P_{t|t}$ . Check that this agrees with the updates given in Wikipedia.

Problem 2. Show that if  $h$  is a monotone heuristic function for a graph search problem with  $h(g) = 0$  (for goal node  $g$ ) then  $h$  is admissible.

Problem 3. Give an example of a graph and an admissible heuristic for that graph where the A\* algorithm finds a path that is longer than the shortest path.

Problem 4. Define monotonicity for grammar heuristic functions. Your definition should guarantee that the sequence of pairs removed from the queue is in order of increasing priority (lower priority items are not added later).

Problem 5. Give a monotone and admissible heuristic for A\* parsing that is computed entirely from the input string and the weights of the productions of the form  $X \rightarrow a$ .